AP Calculus AB
3.4: P. 129 #1, 2, 4, 5, 10, 13, 14, 16, 21, 23-27, 29-31, 33, 36-38

① $V = s^3$

$\frac{dV}{ds} = 3s^2$

② $s(t) = t^2 - 3t + 2 \quad v(t) = 2t - 3 \quad a(t) = 2$

a) $s(5) = 12$ m

b) $\frac{s(5) - s(0)}{5 - 0} = \frac{12 - 2}{5} = 2$ m/sec

c) $v(4) = 5$ m/sec

d) $a(4) = 2$ m/sec^2

e) $v(t) = 2t - 3$

$2t - 3 = 0$

$t = 1.5$ sec

f) $s(t) = t^2 - 3t + 2$

minimum when $t = \frac{3}{2}$

$s(1.5) = -0.25$ m

④ Mars:

$s(t) = 1.86t^2$

$v(t) = 3.72t$

$16.6 = 3.72t$

$t = 4.46$ sec

Jupiter:

$s(t) = 11.44t^2$

$v(t) = 22.88t$

$16.6 = 22.88t$

$t = 0.726$ sec

⑩ $C(x) = 2000 + 100x - 0.1x^2$

a) $C(100) = \$11000.00$

b) $C'(x) = 100 - 0.2x$

$C'(100) = \$80.00$

c) $C(101) - C(100)$

$\$11079.90 - \$11000.00 = \$79.90$

⑤ $s(t) = 24t - 4.9t^2$

$v(t) = 24 - 9.8t$

$24 - 9.8t = 0$

$t \approx 2.44898$ sec

$s(2.44898) \approx 29.3878$ m

⑬ $v(t) = 2t^3 - 9t^2 + 12t - 5$

$a(t) = 6t^2 - 18t + 12$

$6t^2 - 18t + 12 = 0$

$t^2 - 3t + 2 = 0$

$(t - 2)(t - 1) = 0$

$t = 1$ or $t = 2$

$v(1) = 0$ m/sec

$v(2) = -1$ m/sec

Speed at $t = 0$ is 0 m/sec.

Speed at $t = 2$ is 1 m/sec.

(14) $f'(x) = 3x^2$

a) $g'(x) = 3x^2$ $\quad h'(x) = 3x^2$ $\quad t(x) = 3x^2$

b) all three graphs are the same

c) $f(x) = x^3 + c$, where c is a constant

d) yes: $f(x) = x^3$ $\qquad$ e) yes: $f(x) = x^3 + 3$

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(16) a) ~ 190 ft/sec $\quad$ b) 2 sec

c) at 8 seconds (when the velocity was 0 ft/sec)

d) around 11 seconds; falling down at 90 ft/sec

e) about 3 seconds $\qquad$ f) just before the engine stopped (around 2 seconds)

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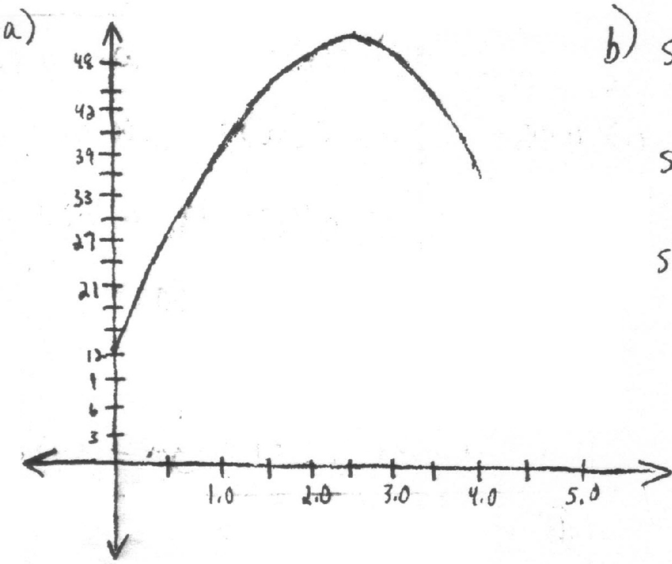
(21) a)



b) $s'(1.0) \approx 18$

$s'(2.5) = 0$

$s'(3.5) = -12$

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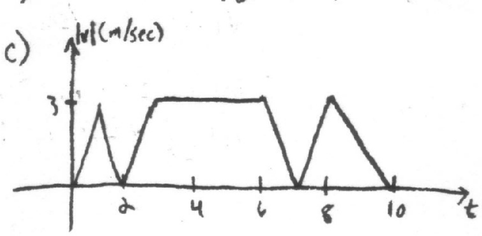
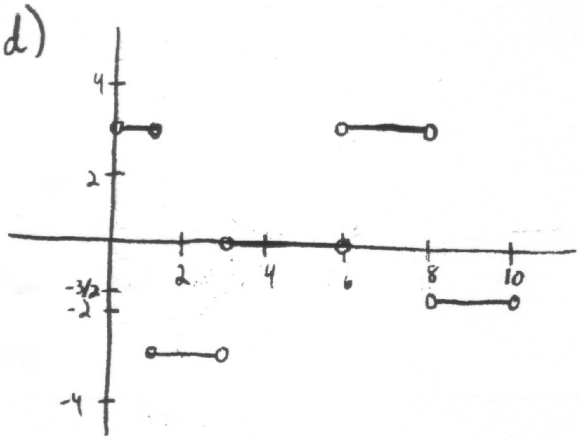
(23) a) at $t = 2$ and $t = 7$ $\qquad$ b) from 3 to 6 seconds

c)



(23) d)

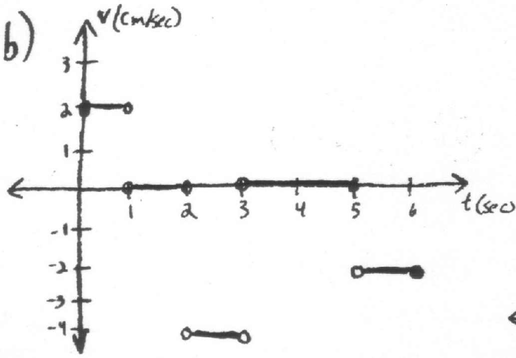


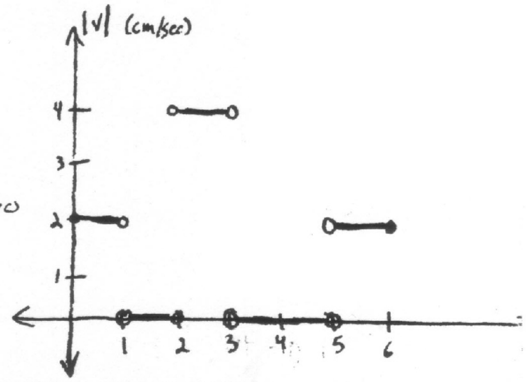
(24) a) left: $(2,3) \cup (5,6)$

right: $[0,1)$

standing still: $(1,2) \cup (3,5)$

b)





(25) a) forward: $[0,1) \cup (5,7)$ backward: $(1,5)$

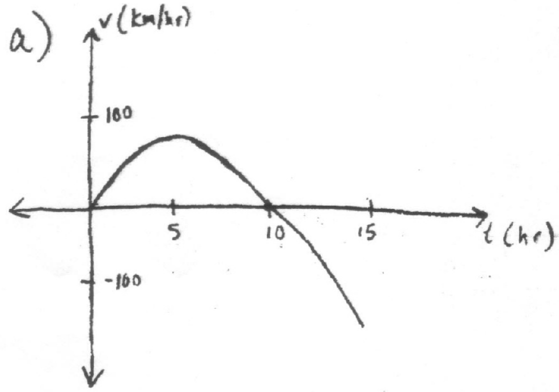
speeds up: $(1,2) \cup (5,6)$ slows down: $[0,1) \cup (6,7)$

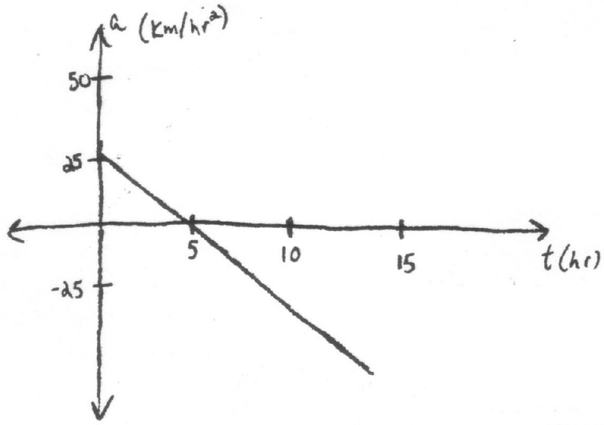
b) positive: $(3,6)$ c) $t=0$

negative: $[0,2) \cup (6,7)$ d) $(7,9]$

zero: $(2,3) \cup (7,9]$

(26) a)





b) $s(t) = 15t^2 - t^3$

$\frac{ds}{dt} = 30t - 3t^2$

$\frac{d^2s}{dt^2} = 30 - 6t$

(27) $s(t) = 490t^2$

$v(t) = 980t$

$a(t) = 980$

a) $160 = 490t^2$

$t \approx 0.57\,\text{sec}$

$V_{ave} = \frac{160}{0.57} \approx 280\ \text{cm/sec.}$

b) $v(0.57) \approx 558.6\ \text{cm/sec}$

$a(0.57) = 980\ \text{cm/sec}^2$

c) $\frac{16}{0.57} \approx 28$ flashes per second

(29) a) $a(t)$

b) $v(t)$

c) $s(t)$

(30) $y = 6\left(1-\frac{t}{12}\right)^2$

$y = 6\left(1-\frac{t}{6}+\frac{t^2}{144}\right)$

a) $\frac{dy}{dt} = 6\left(-\frac{1}{6}+\frac{t}{72}\right) = \frac{t}{12}-1$

b) The fluid level is falling fastest when $\frac{dy}{dt}$ is the most negative, which is when $t=0$ ($\frac{dy}{dt}=-1$)
The fluid level is falling slowest at $t=12$ ($\frac{dy}{dt}=0$)

c) y is decreasing over the entire interval
$\frac{dy}{dt}$ is negative over the entire interval
y is decreasing more rapidly early in the interval, so the magnitude of $\frac{dy}{dt}$ is greater then
y is decreasing less rapidly later in the interval, so the magnitude of $\frac{dy}{dt}$ is less then

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(31) $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dr} = 4\pi r^2$

a) $\frac{dV}{dr} = 16\pi \ ft^3/ft$

b) $\frac{4}{3}\pi(2.2)^3 - \frac{4}{3}\pi(2^3) \approx 11.092$ cubic feet

(33) $s(t) = v_0 t - 16t^2$

$v(t) = v_0 - 32t$

$0 = v_0 - 32t$

$t = \frac{v_0}{32}$

$1900 = v_0\left(\frac{v_0}{32}\right) - 16\left(\frac{v_0}{32}\right)^2$

$v_0 = 348.712 \text{ ft/sec} = 237.758 \text{ mph}$

$$\frac{348.712 \text{ ft}}{1 \text{ sec}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot \frac{1 \text{ miles}}{5280 \text{ ft.}}$$

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(36) a) 135 seconds $\rightarrow$ 2 minutes 15 seconds

b) $\frac{5}{73} \approx 0.068$ furlongs/sec

c) $\frac{4-2}{59-33} = \frac{2}{26} = \frac{1}{13} \approx 0.077$ furlongs/sec

★ use symmetric quantity w/ $t=2$ & $t=4$

d) during the 10^{th} furlong (it only takes 11 seconds to run)

e) during the first furlong

37 a) Assume f is even. (and show $f'(-x) = -f'(x)$)

Then $f'(-x) = \lim\limits_{h\to 0} \dfrac{f(-x+h) - f(-x)}{h}$

$$= \lim_{h\to 0} \frac{f(x-h) - f(x)}{h} \quad \text{(because } f \text{ is even)}$$

Let $k = -h$

$$= \lim_{k\to 0} \frac{f(x+k) - f(k)}{-k}$$

$$= -\lim_{k\to 0} \frac{f(x+k) - f(k)}{k} = -f'(x).$$

b) Assume f is odd. Show $f'(-x) = f'(x)$.

$$f'(-x) = \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h}$$

$$= \lim_{h\to 0} \frac{-f(x-h) + f(x)}{h} \quad \text{(because } f \text{ is odd)}$$

Let $k = -h$

$$= \lim_{k\to 0} \frac{-f(x+k) + f(x)}{k}$$

$$= -\lim_{k\to 0} \frac{f(x+k) - f(x)}{k} = -f'(x)$$

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38 $\dfrac{d}{dx}[fgh] = f \cdot \dfrac{d}{dx}[gh] + gh \cdot \dfrac{d}{dx}[f]$

$$= f \cdot \left[g \cdot \frac{dh}{dx} + h \frac{dg}{dx}\right] + gh \frac{df}{dx}$$

$$\boxed{\frac{d}{dx}[fgh] = fg\frac{dh}{dx} + fh\frac{dg}{dx} + gh\frac{df}{dx}}$$